Introduction to Optimization

1. A company writes adventure books. Let $C(q)$ be the cost of producing q books and $R(q)$ be the revenue of producing q books, both are measured in dollars. The table below gives the company's marginal cost, $\dfrac{dC}{dq}$, and marginal revenue, $\dfrac{dR}{dq}$, measured in dollars per book.

q	500	1000	1500	2000	2500	3000
Marginal Cost	50	40	37	48	56	62
Marginal Revenue	62	60	57	55	52	50

   Assume that $\dfrac{d^2C}{dq^2}$ and $\dfrac{d^2R}{dq^2}$ exist and that the Marginal Cost and Marginal Revenue only increase or only decrease between values in the table.

   The CEO of the company thinks that profit is maximized at 1500 bags, but the CFO thinks that profit is maximized at 2300 bags. Who is more likely to be correct according to the table and why?

2. Suppose f has five zeros and that f' is continuous. How many zeros must f' have? Explain why this is true.

3. Assume that g is twice-differentiable on the closed interval $[1, 5]$ and has only one critical point at $x = 2.5$. Select each statement below that, if true, would by itself be a valid justification that $g(2.5)$ is a local maximum of g on this interval.

   Note: Selecting a statement means it must stand on its own as a justification, without any other assumptions, other than that $x = 2.5$ is a critical point.

   (Select ALL statements that are a valid justification.)

   (a) $g'(2.5) = 0$

   (b) $g''(2.5) = 0$

   (c) $g'(2.5)>0$

   (d) $g''(2.5)>0$

   (e) $g'(2.5)<0$

   (f) $g''(2.5)<0$

   (g) $g'(1)>0$ and $g'(3)<0$

   (h) $g'(1)<0$ and $g'(3)>0$

   (i) $g(1)<g(2.5)<g(5)$

   (j) $g(1)<g(5)<g(2.5)$

4. A baseball outfielder fields a ball and throws it back towards the infield. The baseball's height above the ground $g(t)$ (in feet) t seconds since it was released can be modeled by the function

$$g(t) = -16t^2 + 31.829t + 6.5.$$

By solving which of the following equations can we determine the value of t for which the height of the ball above the ground is maximized?

(a) $g(t) = -16t^2 + 31.829t + 6.5$

(b) $0 = -16t^2 + 31.829t + 6.5$

(c) $g'(t) = -32t + 31.829$

(d) $0 = -32t + 31.829$

(e) $g''(t) = -32$

5. A rectangular pasture is being created with one side along a straight riverbank. The remaining three sides are to be enclosed with a fence. If there are 12 km of fence available, use calculus to find the dimensions of the rectangular pasture to maximize its area.